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Analysis and optimization of duty-cycle in preamble-based random access networks

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Abstract Duty-cycling has been proposed as an effective mechanism for reducing the energy consumption in wireless sensor networks (WSNs). Asynchronous duty-cycle protocols where the receiver wakes up periodically to check whether there is a transmission and the sender transmits preambles to check if the receiver is awake are widely used in WSNs due to the elimination of complex control mechanisms for topology discovery and synchronization. However, the intrinsic simplicity of the asynchronous mechanism has the drawback of smaller energy saving potential that requires the optimization of the duty cycle parameters. In this paper, we propose a novel method for the optimization of the duty-cycle parameters in preamble-based random access networks based on the accurate modeling of delay, reliability and energy consumption as a function of listen time, sleep time, traffic rate and medium access control (MAC) protocol parameters. The challenges for modeling are the random access MAC and the sleep policy of the receivers, which make it impossible to determine the exact time of data packet transmissions, and thus difficult to investigate the performance indicators given by the delay, reliability and energy consumption to successfully receive packets. An analysis of these indicators is developed as a function of the relevant parameters of the network and it is used in the minimization of the energy consumption subject to delay and reliability requirements. The optimization provides significant reduction of the energy consumption compared to the previously proposed protocols in the literature.

Keywords Wireless sensor networks · MAC · IEEE 802.15.4 · Duty cycle · Optimization

1 Introduction

Energy-efficient wireless sensor networks (WSNs) are providing new and affordable services for a variety of applications, including home and industrial automation, health-care monitoring, and smart grids [2–5]. Reducing energy consumption in WSNs requires the design of energy efficient communication protocols since the radio consumes a considerable amount of energy. Idle listening that occurs when the node listens to the radio channel without receiving any packet consumes as much energy as receiving data. Therefore it should be minimized since it does not contribute to the operation of the network. Several duty-cycle protocols have been proposed as an effective mechanism for reducing idle listening [6–8]. The idea is to build a mechanism for receiving and transmitting packets while the nodes cycle between an active state and a sleep state during which energy is saved.

Duty-cycling protocols are of two types: synchronous and asynchronous. In synchronous protocols, nodes coordinate themselves to schedule transmissions (see [8–11]

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and references therein). In asynchronous protocols, such as B-medium access control (MAC) [12] (the standard protocol for TinyOS [13]), X-MAC [14] and λ-MAC [15], the receiver wakes up periodically to check whether there is a transmission, and the sender transmits preambles to check if the receiver is awake. Compared to synchronous duty-cycling protocols, asynchronous protocols have the advantage of not requiring negotiation of the schedule among neighboring nodes to specify when the nodes are awake and asleep [8–11] nor complex control mechanisms for discovering the network topology, keeping the nodes synchronized, and running the schedules efficiently [16, 17]. However, the intrinsic simplicity of the asynchronous mechanism has the drawback of smaller energy saving potential as compared to the more complex synchronous mechanisms, unless the optimization of the duty cycle parameters including listen and sleep times is adapted to the data traffic and network conditions. Moreover, while minimizing energy consumption, such an optimization needs to include the delay and successful packet reception probability (reliability) constraints since many applications require guaranteed arrival of sensor data to the collection center (e.g., security monitoring) and others require a certain degree of reliability in delivering sensor data (e.g., control and automation applications).

The modeling of these three essential metrics including energy consumption, delay to transmit packets, and reliability need to consider the random access of the MAC as it influences remarkably these components [1]. Lowering the duty-cycle implies putting nodes in sleep mode for larger periods. While using a larger sleep time reduces the cost of idle listening at the receiver, it increases the transmission cost as the transmitter uses a longer preamble. Hence, there is a tradeoff between the receiving cost of idle listening and transmission cost of longer preamble. Furthermore, as the sleep time increases, the delay, reliability, and throughput significantly degrade due to the high contention in the medium with increasing traffic during the active time. Energy, delay, and reliability must be characterized analytically as a function of the listen and sleep time of the nodes, the random access, the MAC parameters, the traffic, the network topology, and the wireless channel. Once analytical expressions are available, it is possible to optimize the performance of the duty-cycle protocols. The modelling of the three essential performance metrics and their optimization has been considered in various papers, e.g., [12, 14–29]. However, no paper can be found where the three have been considered altogether, as we survey next.

Energy modelling and its use in sleep time optimization was considered in B-MAC [12] and X-MAC [14]. However, in [12, 14] no delay or reliability constraint on packet delivery is considered, which means that the energy minimization does not guarantee timely successful packet delivery. Moreover, these protocols do not take into account the effect of random access, which is a function of MAC parameters, data traffic, and topology. This is a crucial aspect, since the duration of random access is much larger than the actual packet transmission and may consume significant energy. For example, in IEEE 802.15.4 radios with default parameter settings, the maximum backoff before packet transmission is 27.4 ms whereas the transmission time of a 56 byte packet is 1.79 ms at 250 kbps.

In [18], the authors propose a duty cycle optimization where the objective function is the maximization of the reliability within a deadline subject to a maximum energy consumption constraint. However, a random access MAC is not considered, which prevents the application of these results to random MACs. In [19], the authors present a relevant analysis and some algorithms to select the duty cycle of the nodes in order to minimize the energy consumption and maximize the network lifetime. The study however does not consider the delay and reliability to deliver packets. These assumptions make it difficult to use this approach for random access MAC, where the delay and packet losses could be substantial. In [20–28], the authors pose the problem of determining the optimal duty cycle schedule that minimizes the energy consumption while guaranteeing a maximum delay. However, no random access was considered again, and the packets were assumed to be always successfully received, which implies that the method presented therein cannot be applied to random MAC networks as we discussed before.

In [19], the authors derive the energy consumption of a node as a function of the duty-cycle. This model is then applied to formulate two optimization problems, one minimizing the total energy consumption and the other maximizing the network lifetime. The analytical model of the energy consumption however does not take the collision and contention in sending a packet, the random access mechanism, the packet copy delay, and the delay to tune the transceiver into account. The energy consumption of each node is also calculated for the adaptive duty-cycling of preamble sampling protocol to determine the routing decision in [29]. Each node determines the preferred parent in the routing tree based on the routing cost that is a function of the ratio of the duty-cycles of neighbors to the average duty-cycle in the neighborhood. The duty-cycle is then chosen proportional to the expected number of packets to transmit. This model however does not consider the delay and reliability requirements nor minimizes the energy consumption.

In this paper, we propose a novel method for the optimization of the duty-cycle parameters in WSNs based on the accurate modeling of the three essential performance metrics given by the delay, reliability, and energy consumption as a function of listen time, sleep time, traffic rate and MAC protocol parameters. Such an optimization framework can be inserted to the flexible system architecture of the next generation WSNs such as IDRA [30] for more efficient operation. The original contributions of this paper are two:

- We derive the mathematical expressions of the delay, reliability, and energy consumption of duty-cycling protocols. We then validate these expressions by using Monte Carlo simulations.
- We illustrate the importance of the mathematical expressions of the delay, reliability, and energy consumption of duty-cycling protocols to optimize the duty cycle of the nodes by minimizing the energy consumption under delay and reliability constraints.

The rest of the paper is organized as follows. Section 2 gives the description of the system model. Section 3 describes the framework for the optimization of the duty-cycle parameters of preamble based random access networks. Sections 4, 5 and 6 provide the analytical expressions for delay, reliability and energy consumption respectively. Section 7 gives the implementation details of the duty cycle optimization. Section 8 provides the simulation results. Section 9 gives final remarks.

2 System model

We consider a general WSN topology where nodes transmit packets[1] toward other nodes, and these ones are either sinks, or they route the received packets toward other nodes, as shown in Fig. 1. Such a topology is representative of both clustered and tree networks. In a clustered network, nodes organize themselves into groups or clusters with a node acting as the cluster head. All non-cluster head nodes transmit their packets directly to the cluster head, while the cluster head receives data from all cluster members and transmits them to other cluster heads or a remote sink. Clustered network is an essential topology for a number of standardization groups [31], commercial products and applications [32]. Clustered network topology is supported in networks that require energy efficiency since it allows local data aggregation and eliminates the disadvantages of the unbalanced energy consumption in multi-hop routing and direct transmission to the base station [33, 34]. Analogously, tree networks are of essential interests in many applications. For example, the new routing protocol RPL being standardized from the internet engineering task force, is entirely based on tree topologies [4].

Throughout this paper we consider applications where nodes asynchronously generate packets with rate λ packets per second (see Table 1 for a list of main symbols used in the paper). We consider typical low data rate applications for WSNs for which $\lambda \leq 1$ [18–28]. We explicitly consider the random access mechanism of unslotted IEEE 802.15.4 to improve the delay and reliability performance of preamble sampling protocols. The IEEE 802.15.4 standard has received considerable attention as a low data rate and low power consumption protocol for WSN applications in industry, control, home automation, and health care [3]. It has been adopted with minor variations also by other protocols such as ZigBee [32] and ISA100 [35].

The random access CSMA/CA mechanism of the unslotted IEEE 802.15.4 is as follows: Each node in the network has two variables: *NB* and *BE*. *NB* is the number of times the CSMA/CA algorithm is required to back-off while attempting the current transmission and *BE* is the back-off exponent, which is related to the number of back-off periods a device must wait before it attempts to assess the channel. The parameters that affect random back-off are $BE_{\min}$, $BE_{\max}$ and $NB_{\max}$, which correspond to the minimum and maximum of *BE* and the maximum of *NB*, respectively. We remark that the unslotted IEEE 802.15.4 protocol is not energy efficient since there is no explicit mechanism to save energy consumption. It is natural to combine the duty-cycle mechanism and the unslotted IEEE 802.15.4 protocol.

In preamble sampling protocols, the receiver wakes up periodically for a short time to sample the medium [14]. Such a time is defined as listen time, which we denote by R_l. The time interval between listen times is defined as sleep time, which we denote by R_s. When a sender has data, it transmits a series of short preamble packets, each containing the ID of the target node, until it either receives an acknowledgement packet (ACK) from the receiver or a maximum time given by the sleep plus listen times of the

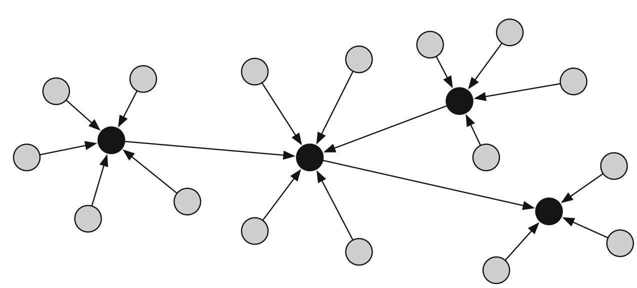

Fig. 1 General network topology, which can be both a clustered or a tree topology. The packets generated by the *gray* nodes are transmitted to the node depicted in the *middle* of each group of nodes. The node can be a sink, or route packets toward other nodes

[1] Throughout this paper, we refer to packets as MAC protocol data units, or MAC frames.

Table 1 Main symbols

Symbol	Meaning
T_1	Random delay spent by the transmitter node before transmitting a preamble packet
T_2	Random delay spent by the transmitter node from the beginning of a transmission until the reception of the acknowledgement
T_3	Random delay spent by the transmitter node from the acknowledgement reception until the transmission of a data packet
T_p	Random delay to wait before a data packet is successfully received
T_s	Random sleep time of the receiver as seen from the transmitter (it is uniformly distributed over [0, R_s])
T_l	Random listen time of the receiver as computed upon the reception of a preamble (it is uniformly distributed over [0, R_l])
T_{hr}	Time employed by the hardware platform to process the packets and transmit them
T_{ack}	Random time before the receiver node can access the channel and send an acknowledgement
$T_{\mathrm{TX,out}}$	Maximum time that a transmitter node waits for an acknowledgement after having sent a preamble
T_{out}	Maximum time that a transmitter node waits from the moment of the reception of an acknowledgement before giving up the data packet transmission
N_p	Maximum number of preambles that can be sent
N_b	Maximum number of back-off to sense the channel for sending a preamble packet
NB	Number of back-off of CSMA/CA
BE	Back-off exponent
$NB_{\max}$	Maximum number of back-offs before declaring a channel access failure
N	Number of nodes in a cluster
λ	Packet generation rate per node
d_{TX}	Probability that a transmitter node has a packet to send in the interval $R_s + R_l$
α	Probability of preamble or acknowledgement packet loss
β	Probability of busy channel
p	Probability of data packet loss
$\phi_{\min}$	Minimum probability of successful packet transmission (reliability requirement)
$\tau_{\max}$	Maximum probability of maximum delay (delay requirement)
E_{tot}	Total energy consumption
$S_{p,k,j}$	The random back-off time before the jth carrier sense for the case where the channel is busy for $k-1$ times and free at the kth time
$\mu_{S_{p,k,j}}$	Average of $S_{p,k,j}$
S_c	Channel sensing duration for clear channel assessment
S_p	Preamble packet duration
S_a	Acknowledgement packet duration
S_d	Data packet duration
S_b	Unit time used by the CSMA/CA algorithm
P_{tx}	Transmit power
P_{rx}	Receive power
P_s	Sleep power
R_s	Sleep time of a receiver node
R_l	Listen time of a receiver node
$\mathcal{A}_k$	Event occurring when the channel is busy for $k-1$ times
$\mathcal{B}_k$	Event occurring when a preamble has to be sent k times before being received in the listen time of the receiver node and the corresponding acknowledgement is sent by the receiver node and received before the time out of the transmitter node
$\mathcal{G}$	Event occurring when a preamble is successfully received during the listen state of the receiver
$\mathcal{H}\mid\mathcal{G}$	Event occurring when the acknowledgement is successfully sent before the time out of the receiver expires provided that a preamble is successfully received
$\mathcal{L}\mid\mathcal{G},\mathcal{H}$	Event occurring when the transmitter sends successfully a data packet provided that a preamble is successfully received and the acknowledgement is also successfully received

receiver is exceeded (see Fig. 2). Following the transmission of each preamble packet, the transmitter node goes in a listen state having a maximum timeout duration $T_{\mathrm{TX,out}}$. If the receiver is the target, it sends an ACK during the pause between the preamble packets. When the receiver node sends an ACK, it waits for data packets for a duration of at least T_{out} even after the end of the wake-up time. Consequently, the maximum listen time is $R_l + T_{\mathrm{out}}$. The extension of T_{out} to the regular listen time allows for the reception of the data packets whose ACK was sent near the expiration of the listen time. Upon reception of the ACK, the sender transmits the data packet to the receiver. However, the transmission of such a packet occurs after sensing an idle channel. If the channel is busy, the data transmission may be delayed too much. The transmitter gives up the transmission of the data packet if the delay from the first attempt to transmit a preamble is larger than $R_s + R_l$.

3 Duty cycle analysis

We believe that the analysis of the duty cycle performance must consider the important metrics given by the delay to transmit a packet, the reliability (or probability to successfully receive a packet) and the average energy consumption employed for the transmissions.

Throughout the paper, for a group of nodes transmitting toward a common receiver node of the group, we denote by $P_d(R_l, R_s, d_{\max})$ the expected probability that the delay of the packet transmissions is less than a certain delay value $d_{\max}$, where the delay is defined as the time interval from the instant the packet is generated until the transmission is successful after receiving the corresponding ACK from the receiver, by $R(R_l, R_s)$ the expected reliability defined as the probability of successful packet reception, and by $E(R_l, R_s)$ the expected total energy consumption to transmit and receive packets. The analysis of the duty cycle requires the investigation of these performance metrics. The availability of the expressions for the performance metrics are of fundamental interest in many situations. For example, in the duty-cycle parameter optimization, $P_d(R_l, R_s, d_{\max})$, $R(R_l, R_s)$, and $E(R_l, R_s)$ can be used to optimally determine the duration of the sleep and listening time of the nodes. In other situations such as routing, next hop node could be chosen by considering the expected delay and reliability required to send a packet as available from analysis [36].

Let us consider in more detail the optimization example. The aim of the duty cycle optimization is to find the listen and sleep time of each receiver node such that the overall energy of the network is minimized under delay and reliability constraints. As a use case of the application of the formulations for $E(R_l, R_s)$, $P_d(R_l, R_s, d_{\max})$, and $R(R_l, R_s)$, the optimization problem is formulated as

$$\min_{R_l, R_s} \quad E(R_l, R_s) \tag{1a}$$
$$\text{s.t.} \quad P_d(R_l, R_s, d_{\max}) \geq \tau_{\max}, \tag{1b}$$
$$R(R_l, R_s) \geq \phi_{\min}. \tag{1c}$$

where $\tau_{\max}$ is the desired probability that the delay is less than $d_{\max}$, and $\phi_{\min}$ is the minimum desired probability with which a data packet should be received. We define $d_{\max}$, $\tau_{\max}$, and $\phi_{\min}$ as the application requirements. The optimization problem (1) is motivated by many applications that require meeting certain delay and reliability requirements ($d_{\max}$, $\tau_{\max}$, and $\phi_{\min}$) for the correct

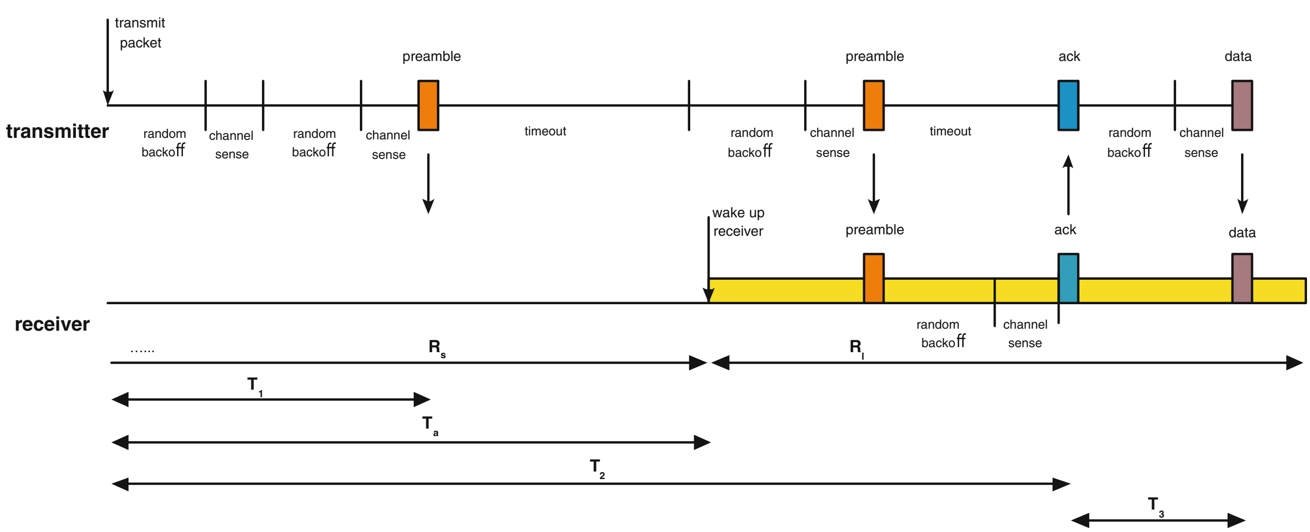

Fig. 2 Communication states between a transmitter and a receiver. A random number of preambles are sent before that one falls in the listen period of the receiver. Afterwards, the receiver sends an ACK. When the transmitter hears the ACK, the data packet is sent. Note that R_l is the listen time and R_s is the sleep time of the receiver node. T_1 is the random delay spent by the transmitter node to complete the transmission of a preamble packet. T_2 is the random delay spent by the transmitter node until the receiver node is in the listen state and an ACK packet reaches the transmitter node. T_3 is the random delay spent by the transmitter node from the instant of ACK reception until the transmission of a data packet. T_a is the random time to wait, from the point of view of the transmitter, from the beginning of the transmission until the start of the listen time

execution of decisions concerning the phenomena sensed [2, 37]. Low delay and high reliability may demand significant energy consumption, therefore delay and reliability must usually be just adequate in meeting the application requirements so as to minimize the energy.

By setting $\tau_{\max} = 0$ and $\phi_{\min} = 0$ in problem (1), one obtains an optimization similar to the one posed in X-MAC [14] (where, in addition, no random access MAC was considered). The optimization problems posed in [20–28] minimize the energy only subject to delay constraints, which is equivalent to the condition $\phi_{\min} = 0$, and the assumption of perfect packet reception probability, which is not acceptable in real MAC. It follows that our formulation is much more general than the optimization of the duty cycle parameters performed in [14, 20–28], where the two constraints have not been considered together.

To solve optimization problems such as (1), or in general understanding performance of duty cycling protocols over random access networks, we need the expressions of the performance metrics including the delay, reliability and energy consumption, which we develop in Sects. 4, 5 and 6 respectively.

4 Delay analysis

In this section, we derive the mathematical expression for the delay experienced for the successful packet transmission from a transmitter node to the receiver given in Eq. (1b). Calculating the delay to successfully transmit a packet requires determining the following delay components (see Fig. 2):

- T_1: random delay spent by the transmitter node to complete the transmission of a preamble packet.
- T_2: random delay spent by the transmitter node until the receiver node is in the listen state and an ACK packet reaches the transmitter node.
- T_3: random delay spent by the transmitter node from the instant of ACK reception until the transmission of a data packet.

The delay to successfully transmit a data packet is then $T_p = T_2 + T_3$. In the following, we find the expressions for these three delay components T_1, T_2, and T_3 in Sects. 4.1 and 4.2, and then the delay probability together with the validation in ns-2 in Sect. 4.3.

4.1 Modeling of T_1

The mechanism to transmit a preamble packet is the same as the one for data packets, for we are assuming to use IEEE 802.15.4. Let β be the busy channel probability. Let N_b be the maximum number of back-offs of a preamble, namely the number of times that the transmitter node attempts to access the channel before giving up the transmission of a preamble. The transmitter may attempt for the transmission of the preamble only once from the application layer, in which case $N_b = NB_{\max}$, or try to transmit the same preamble multiple times until the maximum expiration time equal to R_s therefore $NB_{\max} \le N_b \le R_s/S_c$ where S_c is the sensing time of the channel following backoff.

Let $S_{p,k,j}$ be the random back-off time before the jth carrier sense for the case where the channel is busy for $k-1$ times and free at the kth time. It follows that $S_{p,k,j}$ has a uniform distribution in the interval $[0, (2^{r(j)} - 1)S_b]$,for $j = 1, \ldots, N_b$, where $r(j) = \min(\mathrm{rem}(j, NB_{\max}) + BE_{\min} - 1, BE_{\max})$, with $\mathrm{rem}(\cdot,\cdot)$ being the remainder of the division of the first by the second argument, and $S_b = aUnitback\text{-}off\text{-}Period$ is the back-off period [3]. Denote by $\mathscr{A}_k$ the event where the channel is busy for $k-1$ times and free at the kth time, and by $\mathscr{A}$ the event that a preamble is transmitted with at maximum N_b preambles. Random delay T_1 spent by the transmitter node before transmitting a preamble packet within N_b attempts is then expressed as

$$T_1 = \sum_{k=1}^{N_b}\left[\sum_{j=1}^{k}(S_{p,k,j} + S_c) + T_{\mathrm{hr}}\right]\mathbb{1}_{\mathscr{A}_k|\mathscr{A}} = \sum_{j=1}^{N_b}\Sigma_k \mathbb{1}_{\mathscr{A}_k|\mathscr{A}}\,, \tag{2}$$

where $\mathbb{1}_{(\cdot)}$ is the indicator function (its value is 1 if the argument is true, and 0 otherwise) and $\Sigma_k = \sum_{j=1}^{k}(S_{p,k,j} + S_c) + T_{\mathrm{hr}}$ be the random variable describing the time spent for the kth random back-off, where T_{hr} is the time employed by the hardware platform to process the packets and transmit them.

Lemma 1 *The average and the correlation of* T_1 *are*

$$\mu_{T_1} = \mathbb{E}T_1 = \sum_{k=1}^{N_b}\mu_{\Sigma_k}\frac{\beta^{k-1}}{\sum_{j=1}^{N_b}\beta^{j-1}}\,,$$

$$\rho_{T_1} = \mathbb{E}T_1^2 = \sum_{k=1}^{N_b}\rho_{\Sigma_k}\frac{\beta^{k-1}}{\sum_{j=1}^{N_b}\beta^{j-1}}\,,$$

where $\rho_{\Sigma_k} = \sigma^2_{\Sigma_k} + \mu^2_{\Sigma_k}$ *and*

$$\mu_{\Sigma_k} = \mathbb{E}[\Sigma_k] = \sum_{j=1}^{k}\left[\mu_{S_{p,k,j}} + S_c\right] + T_{\mathrm{hr}},$$

$$\sigma^2_{\Sigma_k} = \mathbb{E}[\Sigma_k - \mathbb{E}\Sigma_k]^2 = \sum_{j=1}^{k}\sigma^2_{S_{p,k,j}}.$$

where $\mu_{S_{p,k,j}}$ *and* $\sigma_{S_{p,k,j}}$ *denote the mean and variance of the random variable* $S_{p,k,j}$ *uniformly distributed in the interval* $[0, (2^{r(j)} - 1)S_b]$.

Proof Since $\mathscr{A}_k$ is the event where the channel is busy for $k-1$ times and free at the kth time, the probability of this event is $\Pr[\mathscr{A}_k] = \beta^{k-1}(1-\beta)$, where β is assumed to be independent at each attempt. Such an assumption has been widely adopted in the literature for both unsaturated traffic (see, e.g., [38–40] and references therein), and saturated traffic, for which is quite accurate [41].

The probability of the event $\mathscr{A}$ that a preamble is transmitted with at maximum N_b preambles is

$$\Pr[\mathscr{A}] = \Pr\left[\sum_{j=1}^{N_b} \mathscr{A}_j\right] = \sum_{j=1}^{N_b} \Pr[\mathscr{A}_j],$$

where the equality comes from that the events $\mathscr{A}_j, j = 1, \ldots, N_b$ are mutually exclusive. It holds

$$\Pr[\mathscr{A}_k|\mathscr{A}] = \frac{\Pr\left[\mathscr{A}_k \sum_{j=1}^{N_b} \mathscr{A}_j\right]}{\Pr[\mathscr{A}]} = \frac{\Pr[\mathscr{A}_k]}{\sum_{j=1}^{N_b} \Pr[\mathscr{A}_j]}$$
$$= \frac{\beta^{k-1}}{\sum_{j=1}^{N_b} \beta^{j-1}}.$$

Notice that Σ_k is given by the sum of independent uniformly distributed random variables plus a constant. Therefore the mean of Σ_k is given by

$$\mu_{\Sigma_k} = \mathbb{E}[\Sigma_k] = \sum_{j=1}^{k}\left[\mu_{S_{p,k,j}} + S_c\right] + T_{\mathrm{hr}}.$$

where $\mu_{S_{p,k,j}} = \left(2^{r(j)} - 1\right)S_b/2$. The variance of Σ_k is given by the sum of the variances of $S_{p,k,j}$ hence

$$\sigma^2_{\Sigma_k} = \mathbb{E}[\Sigma_k - \mathbb{E}\Sigma_k]^2 = \sum_{j=1}^{k} \sigma^2_{S_{p,k,j}}.$$

where $\sigma^2_{Sp,k,j} = (2^{2r(j)} - 1)S_b^2/12$. By using μ_{Σ_k} and $\sigma^2_{\Sigma_k}$ and applying the properties of the expectation operator, the lemma follows. □

Since T_1 is the weighted sum of uniform random variables having different mean and variance, no closed form expression is available for the probability mass function. However, we resort to a Gaussian distribution to approximate the probability mass function of T_1. In Sect. 4.3 we show that this approximation matches well the real one obtained via Monte Carlo simulations. Previous works that have used a Gaussian approximation include [18] and references therein (which, however, do not include random access).

Since the ACK packets are transmitted by following exactly the same mechanism of the preamble packets, the time employed by the receiver node to send an ACK packet upon the reception of a preamble packet can be modeled as done for T_1, the only difference being that N_b must be replaced by $NB_{\max}$, so that T_{ack} is approximated by a Gaussian distribution with average and correlation given by

$$\mu_{T_{\mathrm{ack}}} = \sum_{k=1}^{NB_{\max}} \mu_{\Sigma_k} \frac{\beta^{k-1}}{\sum_{k=1}^{NB_{\max}} \beta^{k-1}},$$
$$\rho_{T_{\mathrm{ack}}} = \sum_{k=1}^{NB_{\max}} \rho_{\Sigma_k} \frac{\beta^{k-1}}{\sum_{k=1}^{NB_{\max}} \beta^{k-1}},$$

and $\sigma^2_{T_{\mathrm{ack}}} \triangleq \rho_{T_{\mathrm{ack}}} - \mu^2_{T_{\mathrm{ack}}}$.

4.2 Modeling of T_2 and T_3

In this section we model T_2, the random delay the transmitter node waits until an ACK is sent by the receiver node and reaches the transmitter node that sent the preamble. With this goal in mind, we define three random variables, T_a, T_l, and N_p, as follows.

First, let us denote by T_a the random time to wait from the beginning of the transmission until the start of the listen time (see Fig. 2). $T_a = 0\mathbb{1}_{\bar{\mathscr{S}}} + T_s\mathbb{1}_{\mathscr{S}}$, where the event $\mathscr{S}$ occurs when the receiver node is sleeping at the beginning of the transmission and T_s is the random time to wait until the receiver wakes up given that the receiver node is sleeping at the beginning of the transmission. Since T_s has a uniform distribution in the range [0, R_s] and a node sleeps for R_s seconds and is awake for R_l seconds, it follows that

$$\Pr[\mathscr{S}] = \frac{R_s}{R_s + R_l}, \qquad \Pr[\bar{\mathscr{S}}] = 1 - \Pr[\mathscr{S}].$$

where $\Pr(T_a = 0) = \Pr[\bar{\mathscr{S}}]$. The probability of T_a for $0 < T_a \le R_s$ is given by

$$\Pr[T_a] = \Pr[T_s]\Pr[\mathscr{S}] = \Pr[T_s]\frac{R_s}{R_s + R_l}$$

Therefore, the probability mass function of T_a is

$$\Pr[T_a] = \begin{cases} \frac{1}{R_s+R_l}, & 0 < T_a \le R_s \\ \frac{R_l}{R_s+R_l}, & T_a = 0 \end{cases}$$

We next define T_l as the time interval from the moment wherein the preamble packet is received in the listen time of the receiver, until the listen time expires. By following the same approach as the one used for the characterization of T_s, it follows that T_l has a uniform distribution in the interval [0, R_l].

Consider N_p, the random number of preambles that should be sent before one falls in the listen time of the receiver and the ACK is sent back by the receiver node. Let us define $\mathscr{B}_k$ as the event that a preamble has to be sent k times before being received in the listen time of the receiver node and the corresponding ACK is sent by the receiver node and received before the time out of the transmitter node. We assume that the event $\mathscr{B}_k$ is conditioned on the random wake-up time T_a of the receiver and on the random remaining listen time T_l of the receiver (see Fig. 2). These times are random from the point of view of the transmitter, which does not know when the receiver wakes up and when it will go to sleep.

We are now in the position of defining the delay T_2:

$$T_2 = \sum_{k=1}^{N_p} \left[\sum_{j=1}^{k} T_{1,k,j} + (k-1)T_{\mathrm{TX,out}} + T_{\mathrm{ack}} \right] \mathbb{1}_{\mathscr{B}_k|\mathscr{B}}, \tag{3}$$

where $T_{1,k,j}$ is the delay of the transmission of the jth preamble when the preamble is sent k times with the distribution given in Eq. (2) and $\mathscr{B}$ is the probability that the transmitter node receives an ACK within N_p preambles.

Lemma 2 *The average and variance of* T_2 *are*

$$\mu_{T_2} = \sum_{k=1}^{N_p} \left[k\mu_{T_1} + (k-1)T_{\mathrm{TX,out}} + \mu_{T_{\mathrm{ack}}} \right] \frac{\Pr[\mathscr{B}_k]}{\sum_{j=1}^{N_p} \Pr[\mathscr{B}_j]},$$
$$\sigma^2_{T_2} = \sum_{k=1}^{N_p} \sigma^2_{T_{2,k}} \frac{\Pr[\mathscr{B}_k]}{\sum_{j=1}^{N_p} \Pr[\mathscr{B}_j]},$$

where $\sigma^2_{T_{2,k}}$ *is the variance of* $\sum_{j=1}^{k} T_{1,k,j} + (k-1)T_{\mathrm{TX,out}} + T_{\mathrm{ack}}$ *and*

$$\begin{aligned}\Pr[\mathscr{B}_k] = {} & (\Pr[\mathscr{C}_k\mathscr{E}_k] - \Pr[\bar{\mathscr{D}}_k])\Pr[T_{\mathrm{ack}} \le T_{\mathrm{TX,out}}](1-\alpha)^2 \\ & + (\Pr[\mathscr{E}_k] - \Pr[\mathscr{C}_k\mathscr{E}_k]) \\ & \times \Pr[T_{\mathrm{ack}} \le T_{\mathrm{TX,out}}]\alpha(1-\alpha)^2 + (\Pr[\mathscr{E}_k] \\ & - \Pr[\mathscr{C}_k\mathscr{E}_k])\big(1 - \Pr[T_{\mathrm{ack}} \le T_{\mathrm{TX,out}}]\big) \\ & \times \Pr[T_{\mathrm{ack}} \le T_{\mathrm{TX,out}}](1-\alpha)^3 + (\Pr[\mathscr{E}_k] \\ & - \Pr[\mathscr{C}_k\mathscr{E}_k])\big(\Pr[T_{\mathrm{ack}} \le T_{\mathrm{TX,out}}]\big)^2\alpha(1-\alpha)^3,\end{aligned}$$

where

$$\Pr[\mathscr{C}_k] = P_1\left(\frac{T_a - (k-2)T_{\mathrm{TX,out}}}{k-1}\right),$$
$$\Pr[\bar{\mathscr{D}}_k] = P_1\left(\frac{T_a - (k-1)T_{\mathrm{TX,out}}}{k}\right),$$
$$\Pr[\mathscr{E}_k] = P_1\left(\frac{T_a + T_l - (k-1)T_{\mathrm{TX,out}}}{k}\right),$$
$$\begin{aligned}\Pr[\mathscr{C}_k\mathscr{E}_k] = {} & \Pr[\mathscr{C}_k]\Pr[T_1 \le T_l - T_{\mathrm{TX,out}}] \\ & + \Pr[\mathscr{E}_k](1 - \Pr[T_1 \le T_l - T_{\mathrm{TX,out}}]).\end{aligned}$$

Proof The proof is based on expressing the average and variance of T_2 in terms of $\Pr[\mathscr{B}_k]$ and then finding the expression for $\Pr[\mathscr{B}_k]$. Based on the expression of T_2 in Eq. 3, its average and variance are expressed as follows:

$$\mu_{T_2} = \sum_{k=1}^{N_p} \left[k\mu_{T_1} + (k-1)T_{\mathrm{TX,out}} + \mu_{T_{\mathrm{ack}}} \right] \Pr[\mathscr{B}_k|\mathscr{B}],$$
$$\sigma^2_{T_2} = \sum_{k=1}^{N_p} \sigma^2_{T_{2,k}} \Pr[\mathscr{B}_k|\mathscr{B}],$$

Using that the events $\mathscr{B}_j, j = 1, \ldots, N_p$ are mutually exclusive, we have

$$\Pr[\mathscr{B}] = \Pr\left[\sum_{k=1}^{N_p} \mathscr{B}_k\right] = \sum_{k=1}^{N_p} \Pr[\mathscr{B}_k],$$

Then

$$\Pr[\mathscr{B}_k|\mathscr{B}] = \frac{\Pr\left[\mathscr{B}_k \sum_{j=1}^{N_p} \mathscr{B}_j\right]}{\Pr[\mathscr{B}]} = \frac{\Pr[\mathscr{B}_k]}{\sum_{j=1}^{N_p} \Pr[\mathscr{B}_j]}.$$

□

We now need to find an expression for $\Pr[\mathscr{B}_k]$. To do that, we use the expression of $\mathscr{B}_k$ in terms of simpler events provided in the following proposition. First, we define by $\mathscr{P}_j$ the event of losing a preamble or an ACK packet at time index j due to bad channel or collisions, and let $\Pr[\mathscr{P}_j] = \alpha$. Such a probability is different from the loss probability of data packets, which recall that we denote by p, because the size of preambles and ACK is much smaller than data packets. We assume that these probabilities are independent at each attempt. Such an approximation has been widely adopted in the literature (see, e.g., [38] and references therein).

Proposition 1 *Let* $\mathscr{B}_k$, *with* $k \in \mathbb{N}$, *the event occurring when* $k-1$ *preambles are sent before the* k*th is received in the listen time of the receiver node, and the ACK is sent back by the receiver node and received before the time out of the transmitter node. Let* Ω *be the certain event. Then*

$$\begin{aligned}\mathscr{B}_k = {} & [\mathscr{C}_k + \mathscr{D}_{k-1}\mathscr{E}_{k-1}\mathscr{P}_1 + \mathscr{D}_{k-1}\mathscr{E}_{k-1}\bar{\mathscr{P}}_1\bar{\mathscr{F}}_{k-1} \\ & + \mathscr{D}_{k-1}\mathscr{E}_{k-1}\bar{\mathscr{P}}_1\mathscr{F}_{k-1}\mathscr{P}_2]\mathscr{D}_k\mathscr{E}_k\mathscr{F}_k\bar{\mathscr{P}}_3\bar{\mathscr{P}}_4,\end{aligned}$$

where

$$\begin{aligned} & \mathscr{C}_k = [(k-1)\mathbb{1}_{k-1\ge 0}T_1 + (k-2)\mathbb{1}_{k-2\ge 0}T_{\mathrm{TX,out}} \le T_a], \\ & \mathscr{D}_0 = \Omega, \qquad \mathscr{D}_k = [kT_1 + (k-1)\mathbb{1}_{k-1\ge 0}T_{\mathrm{TX,out}} > T_a], \\ & \mathscr{E}_k = [kT_1 + (k-1)\mathbb{1}_{k-1\ge 0}T_{\mathrm{TX,out}} \le T_a + T_l], \\ & \bar{\mathscr{F}}_0 = [T_{\mathrm{ack}} > T_{\mathrm{TX,out}}], \quad \bar{\mathscr{F}}_{k-1} = [T_{\mathrm{ack}} > T_{\mathrm{TX,out}}|\mathscr{D}_{k-1}], \\ & \mathscr{F}_k = [T_{\mathrm{ack}} \le T_{\mathrm{TX,out}}|\mathscr{D}_k]. \end{aligned} \tag{4}$$

Proof The proof is by iteration. First, consider the case of $k = 1$. Then, the event of reception of a preamble after the first attempt occurs when the preamble is sent during the listen time of the receiver node, and the ACK is sent before the timeout of the receiver node:

$$\mathscr{B}_1 = [T_1 > T_a][T_1 \le T_a + T_l][T_{\mathrm{ack}} \le T_{\mathrm{TX,out}}]\bar{\mathscr{P}}_1\bar{\mathscr{P}}_2,$$

where recall that $\bar{\mathscr{P}}_1$ is the event that the preamble is successfully received (no channel losses and no collisions), and $\bar{\mathscr{P}}_2$ is the event that the acknowledgement is successfully received (no channel losses and no collisions as well).

Consider the case of $k = 2$. A preamble fails because (1) it was sent during the sleep time of the receiver, or (2) it was sent during the listen time of the receiver but there was a loss due to bad channel or collisions, or (3) it was sent during the listen time of the receiver without loss but the receiver was not able to send back the ACK before the time out of the transmitter, or (4) it was sent during the listen time of the receiver without loss but the transmitted ACK before the time out of the transmitter was collided. Then, a second preamble is sent during the listen time and an ACK is sent back before the time out of the transmitter:

$$\begin{aligned}\mathscr{B}_2 = {} & [(T_1 \le T_a) + (T_1 > T_a)(T_1 \le T_a + T_l)\mathscr{P}_1 \\ & + (T_1 > T_a)(T_1 \le T_a + T_l)\bar{\mathscr{P}}_1. \\ & \times (T_{\text{ack}} > T_{\text{TX,out}} | T_1 > T_a) + (T_1 > T_a) \\ & (T_1 \le T_a + T_l)\bar{\mathscr{P}}_!(T_{\text{ack}} \le T_{\text{TX,out}} | T_1 > T_a)\mathscr{P}_2] \\ & \times \left[T_1 + T_{\text{TX,out}} + T_1 > T_a\right]\left[T_1 + T_{\text{TX,out}}\right. \\ & \left. + T_1 \le T_a + T_l\right] \\ & \times \left[T_{\text{ack}} \le T_{\text{TX,out}} | T_1 + T_{\text{TX,out}} + T_1 > T_a\right]\bar{\mathscr{P}}_3\bar{\mathscr{P}}_4.\end{aligned}$$

It is straightforward to generalize previous expression so to obtain the sought proof. □

We are now in the position to derive $\Pr[\mathscr{B}_k]$. From the proof of Proposition 1, a preamble fails because of four events. It follows that $\mathscr{B}_k = \mathscr{B}_{1,k} + \mathscr{B}_{2,k} + \mathscr{B}_{3,k} + \mathscr{B}_{4,k}$, where $\mathscr{B}_{1,k} = \mathscr{C}_k\mathscr{D}_k\mathscr{E}_k\mathscr{F}_k\bar{\mathscr{P}}_1\bar{\mathscr{P}}_2$ is the event that occurs when a preamble was sent during the sleep time of the receiver; $\mathscr{B}_{2,k} = \mathscr{D}_{k-1}\mathscr{E}_{k-1}\mathscr{D}_k\mathscr{E}_k\mathscr{F}_k\mathscr{P}_1\bar{\mathscr{P}}_2\bar{\mathscr{P}}_3$ is the event that occurs when a preamble was sent during the listen time of the receiver but there was a loss due to bad channel or collisions; $\mathscr{B}_{3,k} = \mathscr{D}_{k-1}\mathscr{E}_{k-1}\bar{\mathscr{F}}_{k-1}\mathscr{D}_k\mathscr{E}_k\mathscr{F}_k\bar{\mathscr{P}}_1\bar{\mathscr{P}}_2\bar{\mathscr{P}}_3$ is the event occurring when the preamble was sent during the listen time of the receiver without loss but the receiver was not able to send back the ACK before the time out of the transmitter; and finally $\mathscr{B}_{4,k} = \mathscr{D}_{k-1}\mathscr{E}_{k-1}\mathscr{F}_{k-1}\mathscr{D}_k\mathscr{E}_k\mathscr{F}_k\mathscr{P}_1\bar{\mathscr{P}}_2\bar{\mathscr{P}}_3\bar{\mathscr{P}}_4$ is the event occurring when the preamble was sent during the listen time of the receiver without loss but the transmitted ACK before the time out of the transmitter was collided. Notice that $\mathscr{B}_{1,k}\mathscr{B}_{2,k} = \emptyset$, $\mathscr{B}_{1,k}\mathscr{B}_{3,k} = \emptyset, \mathscr{B}_{1,k}\mathscr{B}_{4,k} = \emptyset, \mathscr{B}_{2,k}\mathscr{B}_{3,k} = \emptyset, \mathscr{B}_{2,k}\mathscr{B}_{4,k} = \emptyset, \mathscr{B}_{3,k}\mathscr{B}_{4,k} = \emptyset$ and, therefore,

$$\Pr[\mathscr{B}_k] = \Pr[\mathscr{B}_{1,k}] + \Pr[\mathscr{B}_{2,k}] + \Pr[\mathscr{B}_{3,k}] + \Pr[\mathscr{B}_{4,k}]. \tag{5}$$

In the following, the probabilities of $\mathscr{B}_{1,k}, \mathscr{B}_{2,k}, \mathscr{B}_{3,k}$ and $\mathscr{B}_{4,k}$ are computed.

The probability of $\mathscr{B}_{1,k}$ is given by considering that the event $\mathscr{F}_k$ is independent of the others, so that

$$\Pr[\mathscr{B}_{1,k}] = \Pr[\mathscr{C}_k\mathscr{D}_k\mathscr{E}_k]\Pr[\mathscr{F}_k](1-\alpha)^2.$$

From [42], we have

$$\Pr[\mathscr{C}_k\mathscr{E}_k] = \Pr[\mathscr{C}_k\mathscr{D}_k\mathscr{E}_k] + \Pr[\mathscr{C}_k\bar{\mathscr{D}}_k\mathscr{E}_k],$$

and that $\mathscr{C}_k\bar{\mathscr{D}}_k\mathscr{E}_k = \bar{\mathscr{D}}_k\mathscr{E}_k = \bar{\mathscr{D}}_k$, from which it holds

$$\Pr[\mathscr{C}_k\mathscr{D}_k\mathscr{E}_k] = \Pr[\mathscr{C}_k\mathscr{E}_k] - \Pr[\bar{\mathscr{D}}_k]. \tag{6}$$

Rewriting $\mathscr{C}_k = [kT_1 + (k-1)T_{\text{TX,out}} \le T_a + T_1 + T_{\text{TX,out}}]$, we achieve

$$\mathscr{C}_k\mathscr{E}_k = \begin{cases} \mathscr{E}_k & \text{if } T_a + T_l \le T_a + T_1 + T_{\text{TX,out}} \\ \mathscr{C}_k & \text{otherwise} \end{cases} \tag{7}$$

whereby

$$\begin{aligned}\Pr[\mathscr{C}_k\mathscr{E}_k] = {} & \Pr[\mathscr{C}_k]\Pr[T_1 \le T_l - T_{\text{TX,out}}] + \Pr[\mathscr{E}_k](1 \\ & - \Pr[T_1 \le T_l - T_{\text{TX,out}}]).\end{aligned}$$

This equation and Eq. (6) provide us

$$\Pr[\mathscr{B}_{1,k}] = (\Pr[\mathscr{C}_k\mathscr{E}_k] - \Pr[\bar{\mathscr{D}}_k])\Pr[T_{\text{ack}} \le T_{\text{TX,out}}](1-\alpha)^2, \tag{8}$$

where $\Pr[\mathscr{F}_k] = \Pr[T_{\text{ack}} \le T_{\text{TX,out}}]$.

To compute $\Pr[\mathscr{B}_{2,k}]$, observe that

$$\mathscr{D}_{k-1}\mathscr{E}_{k-1}\mathscr{D}_k\mathscr{E}_k\mathscr{F}_k\alpha(1-\alpha)^2 = \mathscr{D}_{k-1}\mathscr{E}_k\mathscr{F}_k\alpha(1-\alpha)^2,$$

because $\mathscr{D}_k\mathscr{D}_{k-1} = \mathscr{D}_{k-1} = \bar{\mathscr{C}}_k$ and $\mathscr{E}_k\mathscr{E}_{k-1} = \mathscr{E}_k$. It follows that

$$\begin{aligned}\Pr[\mathscr{B}_{2,k}] &= \Pr[\bar{\mathscr{C}}_k\mathscr{E}_k]\Pr[\mathscr{F}_k]\alpha(1-\alpha)^2 \\ &= (\Pr[\mathscr{E}_k] - \Pr[\mathscr{C}_k\mathscr{E}_k])\Pr[T_{\text{ack}} \le T_{\text{TX,out}}]\alpha(1-\alpha)^2.\end{aligned} \tag{9}$$

To compute $\Pr[\mathscr{B}_{3,k}]$, observe that

$$\begin{aligned}& \mathscr{D}_{k-1}\mathscr{E}_{k-1}\bar{\mathscr{F}}_{k-1}\mathscr{D}_k\mathscr{E}_k\mathscr{F}_k(1-\alpha)^3 \\ & \quad = \bar{\mathscr{F}}_{k-1}\mathscr{D}_{k-1}\mathscr{E}_k\mathscr{F}_k(1-\alpha)^3,\end{aligned}$$

because $\mathscr{D}_k\mathscr{D}_{k-1} = \mathscr{D}_{k-1} = \bar{\mathscr{C}}_k$ and $\mathscr{E}_k\mathscr{E}_{k-1} = \mathscr{E}_k$. Note that $\bar{\mathscr{F}}_{k-1}$ and $\mathscr{F}_k$ are independent, and that $\Pr[\bar{\mathscr{F}}_{k-1}] = 1 - \Pr[T_{\text{ack}} \le T_{\text{TX,out}}]$, so

$$\begin{aligned}\Pr[\mathscr{B}_{3,k}] &= \Pr[\bar{\mathscr{C}}_k\mathscr{E}_k]\Pr[\bar{\mathscr{F}}_{k-1}]\Pr[\mathscr{F}_k](1-\alpha)^3 \\ &= (\Pr[\mathscr{E}_k] - \Pr[\mathscr{C}_k\mathscr{E}_k])(1 - \Pr[T_{\text{ack}} \le T_{\text{TX,out}}]) \\ &\quad \times \Pr[T_{\text{ack}} \le T_{\text{TX,out}}](1-\alpha)^3.\end{aligned} \tag{10}$$

To compute $\Pr[\mathscr{B}_{4,k}]$, observe that

$$\begin{aligned}& \mathscr{D}_{k-1}\mathscr{E}_{k-1}\mathscr{F}_{k-1}\mathscr{D}_k\mathscr{E}_k\mathscr{F}_k\alpha(1-\alpha)^3 \\ & \quad = \mathscr{F}_{k-1}\mathscr{D}_{k-1}\mathscr{E}_k\mathscr{F}_k\alpha(1-\alpha)^3,\end{aligned}$$

because $\mathscr{D}_k\mathscr{D}_{k-1} = \mathscr{D}_{k-1} = \bar{\mathscr{C}}_k$ and $\mathscr{E}_k\mathscr{E}_{k-1} = \mathscr{E}_k$. Note that $\mathscr{F}_{k-1}$ and $\mathscr{F}_k$ are independent, so

$$\Pr[\mathscr{B}_{4,k}] = \Pr[\bar{\mathscr{C}}_k \mathscr{E}_k] \Pr[\mathscr{F}_{k-1}] \Pr[\mathscr{F}_k] \alpha(1-\alpha)^3$$
$$= (\Pr[\mathscr{E}_k] - \Pr[\mathscr{C}_k \mathscr{E}_k]) (\Pr[T_{\text{ack}} \le T_{\text{TX,out}}])^2 \alpha(1-\alpha)^3. \tag{11}$$

By putting together (5)–(11), $\Pr[\mathscr{B}_k]$ is calculated.

Since T_2 is given by the weighted sum of variables that we approximated in Sect. 4.1 as Gaussian distributed, it follows that the probability mass function of T_2 can be approximated by a Gaussian random variable.

The delay to successfully send a data packet is given by $T_p = T_2 + T_3$, given T_a and T_l. Looking at Fig. 2, since the transmission of the data packet and the acknowledgement follow the same random access strategy, it is straightforward to see that T_3 can be characterized using the expression calculated for T_{ack} except for a higher constant transmission time of the data packet instead of ACK within T_{hr} therefore T_3 is again approximated by a Gaussian random variable. Finally, T_p is approximated by a Gaussian distribution as well, with average $\mu_{T_p} = \mu_{T_2} + \mu_{T_3}$, and variance $\sigma^2_{T_p} = \sigma^2_{T_2} + \sigma^2_{T_3}$.

4.3 Delay probability

The distribution of the delay we have modeled so far is conditioned on the wake-up time T_a of the receiver and the time interval T_l from preamble reception in the listen time of the receiver until the listen time expires. Therefore, the probability that a packet is delayed some $d_{\max}$ seconds and falls in the listen time of the receiver node is given by

$$P_d(R_l, R_s, d_{\max}) \triangleq \mathbb{E}_{T_a,T_l} \Pr[(T_p \le d_{\max})], \tag{12}$$

where $\mathbb{E}_{T_a,T_l}$ denotes the statistical average with respect to the distribution of T_a and T_l. Since the CDF of T_p is given by a cumulative Gaussian distribution, it is a highly nonlinear function of the random variables T_a and T_l. Therefore, the average $\mathbb{E}_{T_a,T_l}$ is obtained by replacing T_a and T_l with their respective expectations, as proposed in [43, p. 428]. This is equivalent to replacing μ_{T_p} with the easy-to-compute $\mathbb{E}_{T_a,T_l}\mu_{T_p}$ and $\sigma^2_{T_p}$ with $\mathbb{E}_{T_a,T_l}\sigma^2_{T_p}$ in the argument of the cumulative Gaussian distribution of T_p. In the notation adopted for (12), we remarked that the reliability depends on the listen and sleep times R_l and R_s and the maximum desired delay $d_{\max}$.

We validated the analysis by comparing the expectation and variance of (12) to extensive Monte Carlo simulations obtained by an ns-2 simulator. Ns-2 simulator has been shown to provide realistic performance in many papers on duty-cycling for sensor networks [20–26]. The simulator reproduced the network where transmitter nodes send packets according to the preamble-based MAC. All the numerical values set for the simulations are taken coherently with the IEEE 802.15.4 standard with the default MAC parameters [3, 44]. The results were computed by running simulations reproducing 20,000 s of real time, and five simulations were run to remove the dependence on the initial seed of the random generators.

In Lemma 2, we derived the event $\mathscr{B}_k$ for a single transmitter-receiver pair, which means that the expectation and variance of (12) are for a single transmitter-receiver pair. However, the analysis considers explicitly the loss and busy channel probabilities, which accounts for the case of multiple transmitters. Therefore, we consider $\mathscr{B}_k$, the expectation and the variance of (12) as an approximation for the general case of several transmitters. As a matter of fact, we observed good matching between the analysis and the simulations for all cases of practical interest. Figure 3 shows the average delay for different listen and sleep times, different number of nodes and with hidden node terminals. We chose a traffic period larger than 10 s since higher traffic rates exhibit packet loss probabilities larger than 50 %, which is not of practical interest. A good linear relationship between delay and sleep time can be inferred from the Monte Carlo simulations since the packet transmission time and wake time are very short compared to the sleep time. This approximation is valid only when $R_s \ge R_l$ for all the cases considered. However, this is not a limitation, because, to save energy, sensors have to use duty cycles much smaller than 50 %, which is perfectly compatible with $R_s \ge R_l$. It is interesting to remark that the average delay variation with respect to the listen and sleep time of Fig. 3 shows a trend that is similar to the results reported in [21]. Figure 4 shows the variance of the delay. The analysis gives a matching less accurate than the

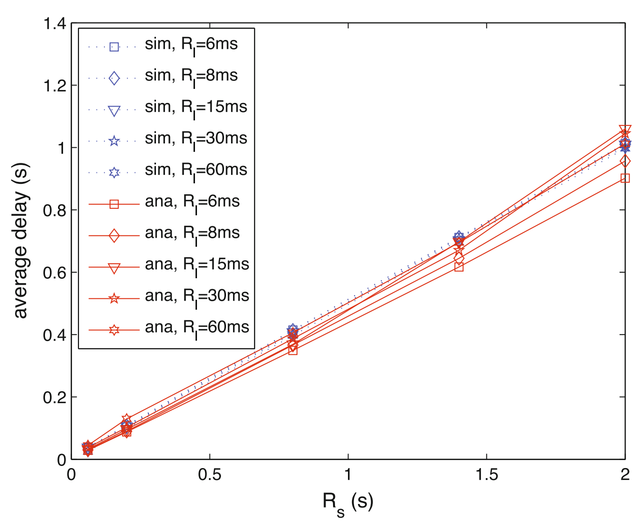

Fig. 3 Average delay to send successfully a data packet as obtained by analysis and simulations for a network with 8 nodes and traffic period 1/λ = 30 s as a function of the sleep time R_s for different values of the listen time R_l

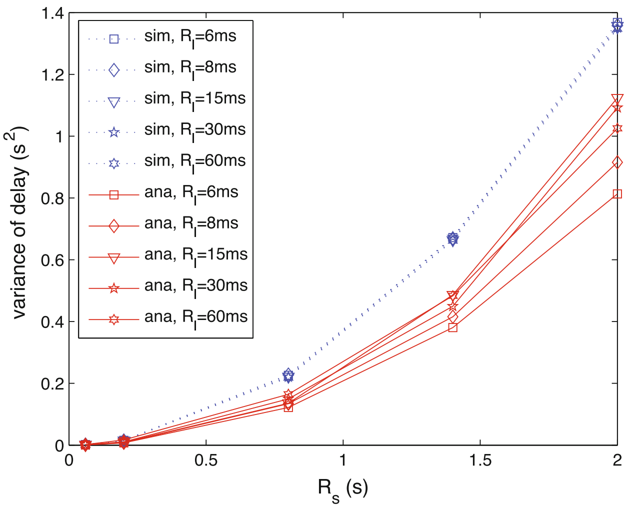

Fig. 4 Variance of the delay to send successfully a data packet $\mathbb{E}_{T_a,T_l}\sigma^2_{T_p}$ as obtained by analysis and simulations as a function of the sleep time R_s for different values of the listen time and for a network with 8 nodes and traffic period 30 s

average delay, but we will see in the next sections that the achieved accuracy is satisfactory for optimization purposes. We demonstrate in [45] that the average and variance of the delay exhibit the same level of matching for different number of nodes, with hidden node terminals, and different traffic generation rates.

5 Reliability analysis

In this section, we analyze the reliability, i.e. probability that a data packet is successfully received. The failure of a data packet transmission is owed to three possibilities: (1) a preamble is not successfully received, (2) the ACK is not successfully received, and (3) the data packet is not successfully received. In the following, we characterize these events.

Let the event $\mathscr{G}$ occur when a preamble is successfully transmitted during the listen time of the receiver within given N_p trials (which occurs with probability $\Pr[\mathscr{N}_p] = 1 - \beta^{N_p}$) and the corresponding ACK is successfully sent within given $NB_{\max}$ trials (which occurs with probability $\Pr[\mathscr{N}_a] = 1 - \beta^{NB_{\max}}$). Then $\mathscr{G}|\mathscr{N}_p\mathscr{N}_a = \sum_{k=1}^{N_p}\mathscr{B}_k$. By observing that $\mathscr{B}_i$ and $\mathscr{B}_j$ are mutually exclusive if $i \neq j$, it follows $\Pr[\mathscr{G}] = (1-\beta^{N_p})(1-\beta^{NB_{\max}})\sum_{k=1}^{N_p}\Pr[\mathscr{B}_k]$.

Define the event $\mathscr{L}|\mathscr{G}$, which occurs when the transmitter sends a data packet successfully, provided that a preamble is successfully received and the ACK is also successfully received, then $\Pr[\mathscr{L}|\mathscr{G}] = \left(1-\beta^{NB_{\max}}\right)(1-p)$, where recall that p is the data packet loss probability.

Finally, by considering the expressions of $\Pr[\mathscr{G}]$ and $\Pr[\mathscr{L}|\mathscr{G}]$, and averaging with respect to the distribution of T_a and T_l, the reliability is given by

$$R(R_l, R_s) = \mathbb{E}_{T_a,T_l}\Pr[\mathscr{G}]\Pr[\mathscr{L}|\mathscr{G}]. \tag{13}$$

The same procedure is followed in calculating these expectations as Eq. (12). In the notation, we remarked that the reliability depends on the listen and sleep times R_l and R_s.

We validated the analysis of the reliability by comparing Eq. (13) to the results of the extensive Monte Carlo simulations as described in Sect. 4.3. Figure 5 shows an example of such simulations as a function of different listen times for the case of 8 nodes and several traffic rates. We see that the analysis follows quite well the simulations results. The good approximation is also confirmed for other choices of the network parameters as illustrated in Figs. 6 and 7. Observe that the reliability decreases as the number of nodes and packet generation rate increase. As the sleep time increases, the expected number of preambles increases, which in turn increases the total traffic. This decreases the network reliability by the resulting increase of collisions and random back-off. Figure 8 illustrates the impact of hidden nodes on reliability. We see that the reliability decreases much faster as the number of hidden nodes increases. In general, we observed that the difference between analysis and simulations is always below 5 %, so we conclude that Eq. (13) is quite a good approximation.

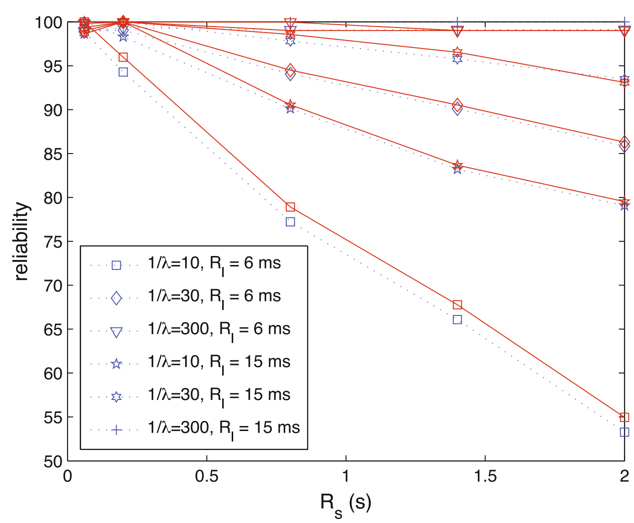

Fig. 5 Reliability as obtained by Eq. (13) and simulations for a network with 8 nodes and traffic period $1/\lambda = 10, 30, 300$ s as a function of the sleep time R_s and for different values of the listen time R_l. The *solid curves* are referred to the analytical results, whereas the *dotted curves* to the simulations. A *dotted curve* and a *solid curve* refer to the same scenario if they have the same marker

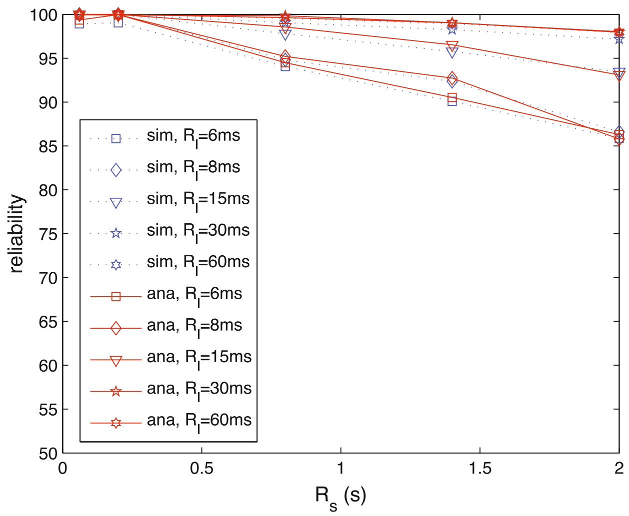

Fig. 6 Reliability as obtained by Eq. (13) and simulations for a network with 8 nodes and traffic period $1/\lambda = 30$ s as a function of the sleep time R_s and for different values of the listen time R_l

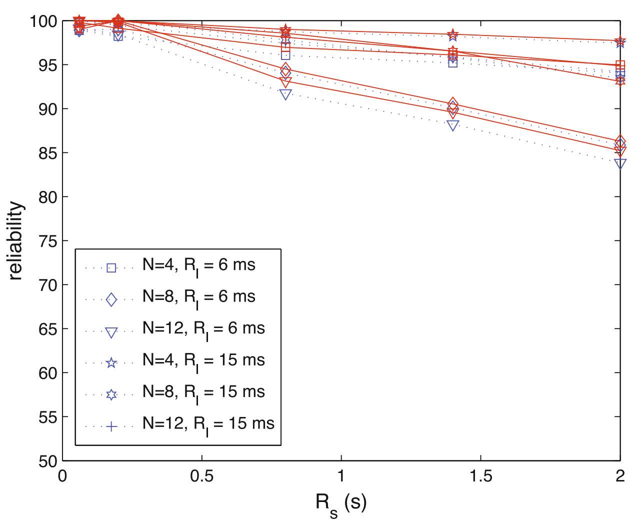

Fig. 7 Reliability as obtained by Eq. (13) and simulations for a network with 4, 8, and 12 nodes and traffic period $1/\lambda = 30$ s as a function of the sleep time R_s and for different values of the listen time R_l. The *solid curves* are referred to the analytical results, whereas the *dotted curves* to the simulations. A *dotted curve* and a *solid curve* refer to the same scenario if they have the same marker

6 Energy consumption analysis

The total normalized energy consumption of the network is given by

$$\mathbb{E}E_{\text{tot}} = \frac{d_{\text{TX}}\mathbb{E}E_{\text{tx}} + \mathbb{E}E_{\text{rx}}}{R_s + R_l}, \tag{14}$$

where $d_{\text{TX}} = 1 - e^{-\lambda(R_s+R_l)}$ is the probability that a transmitter node has at least one data packet to send during the time $R_s + R_l, \mathbb{E}\,E_{\text{tx}}$ is the energy spent by a transmitter

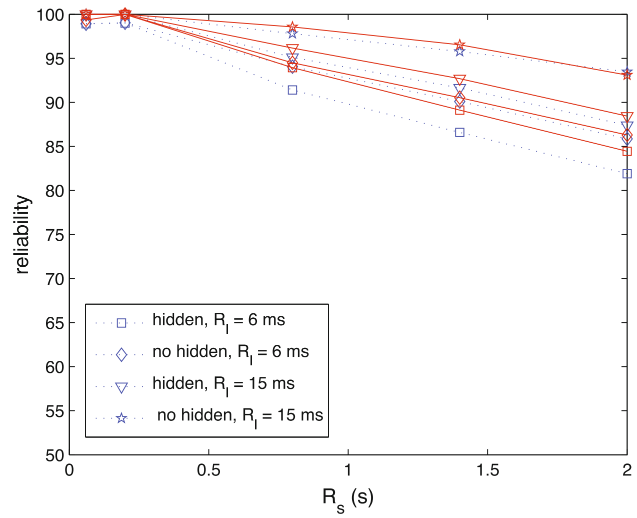

Fig. 8 Reliability as obtained by Eq. (13) and simulations for a network with 8 nodes and traffic period $1/\lambda = 30$ s as a function of the sleep time R_s for different values of the listen time R_l with and without hidden node terminal. The *solid curves* are referred to the analytical results, whereas the *dotted curves* to the simulations. A *dotted curve* and a *solid curve* refer to the same scenario if they have the same marker

node to send a data packet and $\mathbb{E}\,E_{\text{rx}}$ is the energy spent by the receiver node. The total energy consumption is normalized by $R_s + R_l$ to obtain energy per unit time. The energy components $\mathbb{E}\,E_{\text{tx}}$ and $\mathbb{E}\,E_{\text{rx}}$ are characterized next.

Lemma 3 *The instantaneous transmit energy for a transmitter node is upper bounded as*

$$\begin{aligned} E_{\text{tx}} \leq & \sum_{i=1}^{N_p}\left[\sum_{n=1}^{i} E^{(n)}_{\text{tx},T_1} + (i-1)E_{T_{\text{TX,out}}} + E_{\text{tx},T_{\text{ack}}} + E_{\text{tx},T_{\text{data}}}\right]\mathbb{1}_{\mathscr{B}_i} \\ & + \left[\sum_{n=1}^{N_p} E^{n}_{\text{tx},T_1} + (N_p-1)E_{T_{\text{TX,out}}} + E_{\text{tx},T_{\text{ack}}}\right]\mathbb{1}_{\bar{\mathscr{B}}} \end{aligned} \tag{15}$$

where $\bar{\mathscr{A}} = \bigcap_{i=1}^{N_b}\bar{\mathscr{A}}_i, \bar{\mathscr{B}} = \bigcap_{i=1}^{N_p}\bar{\mathscr{B}}_i$ *and*

$$E^{(n)}_{\text{tx},T_1} = \sum_{k=1}^{N_b}\left[\sum_{j=1}^{k}(P_s S_{p,k,j} + P_{\text{rx}} S_c) + P_{\text{tx}} S_p\right]\mathbb{1}_{\mathscr{A}_k}, \tag{16}$$

$$E_{T_{\text{TX,out}}} = T_{\text{TX,out}} P_{\text{rx}}, \tag{17}$$

$$E_{\text{tx},T_{\text{ack}}} = \sum_{k=1}^{NB_{\max}}\left[\sum_{j=1}^{k}(S_{p,k,j} + S_c) + S_a\right] P_{\text{rx}}\mathbb{1}_{\mathscr{A}_k}, \tag{18}$$

$$E_{\mathrm{tx},T_{\mathrm{data}}} = \sum_{k=1}^{NB_{\max}} \left[\sum_{j=1}^{k} (P_s S_{p,k,j} + P_{\mathrm{rx}} S_c) + P_{\mathrm{tx}} S_d \right] \mathbb{1}_{\mathscr{A}_k} + \left[\sum_{j=1}^{NB_{\max}} (P_s S_{p,NB_{\max},j} + P_{\mathrm{rx}} S_c) \right] \mathbb{1}_{\bar{\mathscr{A}}} . \tag{19}$$

where and P_s are the power consumed in transmit, receive and sleep states respectively, S_c is the channel sensing duration for clear channel assessment, S_p is the preamble packet duration, S_a is the ACK duration and S_d is the data packet duration.

Proof The upper bound on E_{tx} is the sum of two main components: one corresponding to the energy spent when the data packet transmission is successful, and the other corresponding to the energy spent when the data packet transmission is not successful. The ith term in the first component corresponds to the case where the ith preamble is successfully transmitted in the listen time of the receiver, and the corresponding ACK is received, i.e. $\mathbb{1}_{\mathscr{B}_i} = 1$ The energy spent for the transmission is then given by the sum of the energy spent in accessing the channel from the first until the ith preamble, i.e. $\sum_{n=1}^{i} E^{(n)}_{\mathrm{tx},T_1}$, listening during the time-out periods for the first $i-1$ preambles, i.e. $(i-1)E_{T_{\mathrm{TX,out}}}$, receiving the following ACK packet (E_{tx}, T_{ack}) and transmitting the data packet (E_{tx}, T_{data}). The second component corresponds to the case where the preamble is not transmitted, and/or the corresponding ACK is not received, i.e. $\mathbb{1}_{\mathscr{B}_i} = 1$. The upper bound on this case is calculated by considering the case where N_p preambles have been sent but none of them were successful.

E_{tx}, $T_1^{(n)}$ given in Eq. (16) is the energy spent during backoff, channel sensing and transmission of the nth preamble. Each term of the sum corresponds to the case where the channel is sensed busy $k-1$ times and sensed idle at the kth trial and includes the energy spent during backoff, i.e. $\sum_{j=1}^{k} P_s S_{p,k,j}$, sensing, i.e. $\sum_{j=1}^{k} P_{rx} S_c$, and transmission of the preamble, i.e. P_{tx} S_p. $E_{T_{\mathrm{TX,out}}}$ given in Eq. (17) is the energy spent by the transmitter during the time out. E_{tx}, T_{ack} given in Eq. (18) is the energy spent by the transmitter while waiting for the ACK. It is derived by following the same approach used for Eq. (16). E_{tx}, T_{data} given in Eq. (19) is the energy spent by the transmitter during the attempt to send the data packet. The first term in Eq. (19) is derived by following the same approach used for (16) whereas the last term corresponds to the case where the packet is not sent since the channel is busy. □

Remark 1 The upper bound in Eq. (15) is derived for analytical tractability by considering the maximum number of preambles to be transmitted when no idle channel is found. The expectation is then calculated by using the expression for $\Pr(\mathscr{B}_i), \Pr(\mathscr{A}_k)$ and expectations of the delay components calculated in Sect. 4. We see at the end of this section that such a bound is reasonable.

The energy consumed at the receiver is upper bounded by

$$\mathbb{E}\, E_{\mathrm{rx}} \le R_s P_s + (R_l + T_{\mathrm{out}}) \max(P_{\mathrm{tx}}, P_{\mathrm{rx}}). \tag{20}$$

where we considered that the receiver can be listening for an additiona time T_{out} after the end of the listen time if an ACK was sent just before the end of the listen time. The upper bound for $\mathbb{E}\, E_{\mathrm{rx}}$ is motivated by the difficulty to provide a closed form expression. The receive energy consists of the energy consumed in idle listening, sending an ACK and receiving data packets. Since these events are highly cross correlated among them and among different sensors, an accurate characterization would require modeling the probability that the receiver node is busy with the reception of a data packet while some other node is trying to send another data packet, which is very difficult to model.

The average total energy consumed as a function of listen and sleep time is then given by

$$E(R_l, R_s) \triangleq \mathbb{E}_{T_a,T_l} \mathbb{E}\, E_{\mathrm{tot}}. \tag{21}$$

as done in Eq. (12).

We validated the analysis of the average energy consumption by comparing Eq. (21) to the results of the extensive Monte Carlo simulations as described in Sect. 4.3.

We validated the analysis of the average energy consumption by comparing the upper bound given by Eq. (21) to Monte Carlo ns-2 simulations, which were obtained as

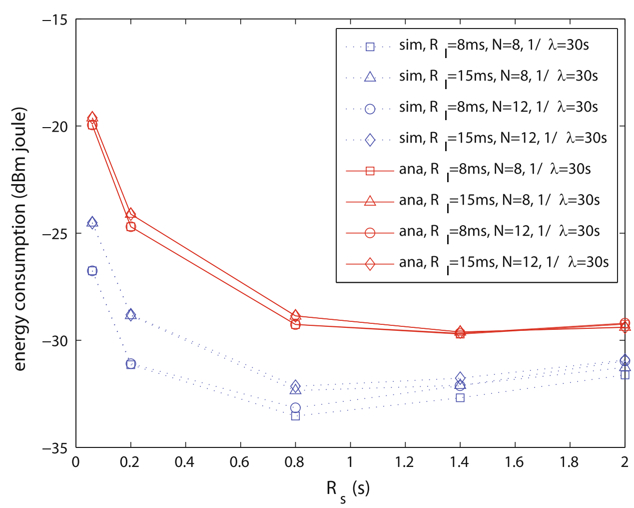

Fig. 9 Average energy consumption as obtained by Eq. (21) and simulations for a traffic rate of $1/\lambda = 30$ s and 8, 12 nodes as a function of the sleep time R_s for different values of the listen time $R_l = 8$, 15 ms

described in Sect. 4.3. Figure 9 reports the analytical model and simulation results of the energy as a function of R_s and R_l for packet generation periods of 30 s. The figure shows a good matching of the upper bound with the simulations. The same conclusion holds for other choices of the network parameters (number of nodes and traffic generation rate), in [45].

7 Duty-cycle optimization

In the previous sections, we have modeled the distributions of the delay to send a data packet from the transmitter to the receiver, the reliability and the energy consumption. Given a set of loss and busy channel probabilities, these expressions can be used off-line to select the optimal values of the listen time R_l and sleep time R_s that minimize the energy consumption given the delay and reliability constraints as provided in problem (1). Since the expressions derived for delay, reliability and energy consumption are highly non linear functions in R_l and R_s, the solution need to be achieved numerically by the methods of non-linear optimization [46, 47]. This computation is not a burden for processors with normal computational constraints. Hence, the solution can be achieved off-line and stored in a look-up table as function of the network topology, channel condition, traffic, and number of nodes. From the cluster-head point of view, these factors are summarized by the loss and busy channel probabilities α and β. Therefore, given these probabilities, the optimal solution of problem (1), denoted by $R_l^*(\alpha, \beta)$ and $R_s^*(\alpha, \beta)$, can be loaded in a look up table to be stored in the cluster-head node. The table is a matrix with rows associated to the set of representative values of α and columns associated to the representative values of β. The cluster-head node can easily do an estimation of the loss probability $\hat{\alpha}$ and busy channel probability $\hat{\beta}$, and read from the look-up table the entries $R_l^*(\alpha, \beta)$ and $R_s^*(\alpha, \beta)$ at location α, β closest to $\hat{\alpha}$ and $\hat{\beta}$ in an Euclidean sense. If we consider 10 values for $\hat{\alpha}$ and 10 for $\hat{\beta}$, the table would have 100 entries. If each entry takes 1 byte, the table has the size of just 0.1 kb.

8 Simulation results

In the previous sections, we investigated the important performance metrics given by delay, reliability and energy consumption and illustrated the validity of the analysis by Monte Carlo simulations. In this section, we further illustrate by simulations the advantage of using the analysis by optimizing the duty cycle parameters.

All the numerical values set for the simulations are taken coherently with the IEEE 802.15.4 standard and the Tmote sensors. In the simulator, transmitter nodes send packets by a preamble-based IEEE 802.15.4 MAC. The receiver node employed a look-up table as described in Sect. 7 to optimize the duty cycle. In particular, during the simulations, the receiver estimated the loss and busy channel probabilities, and then read the look-up table at a location with loss and busy probabilities closest in a Euclidean distance sense to the ones estimated. We considered 5 representative values of the loss probability and 5 representative values for the busy channel probability. Each simulation result was computed by running 5 independent simulations to reproduce 20,000 s of real time.

8.1 Constrained optimization example

We validate our optimization by considering the delay and reliability requirements. In the following, we report the simulation results with optimal listen and sleep time as obtained by the solution of problem (1) as obtained by the method described in Sect. 7 for various cases of application requirements (Fig. 10).

Figure 11 shows the average delay as a function of the reliability requirement $\phi_{\min}$ = 93, 96, 99 %, delay requirement $d_{\max}$ = 0.2, 0.4, 0.6, 0.8 s, $\tau_{\max}$ = 95 %. The average delay is smaller than the delay requirement as required confirming the validity of the optimization. The average delay decreases as the reliability constraint becomes strict, $\phi_{\min}$ = 99 %, due to the fact that the sleep time decreases as the reliability constraint increases. Furthermore, the strict reliability requirement $\phi_{\min}$ = 99 % determines that the dominant constraint of the optimization problem is the reliability, whereas the delay requirement $d_{\max}$ = 0.2, 0.4, 0.6, 0.8 s makes the delay constraint inactive.

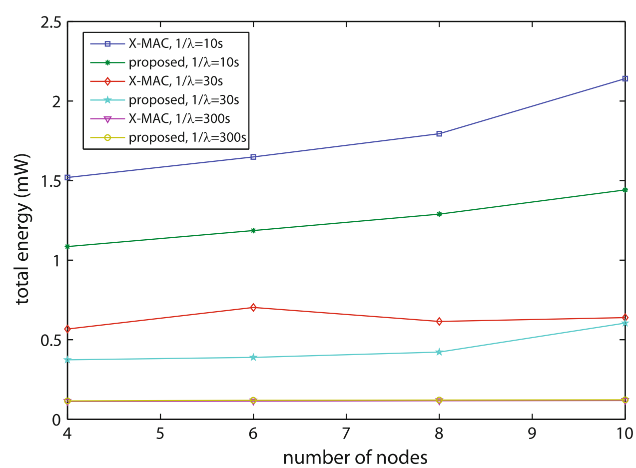

Fig. 10 Comparison of energy consumption as obtained by the optimization of the listen and sleep time proposed by X-MAC [14] and the new optimization subject to reliability and delay constraints of problem (1) as given by the method of Sect. 7

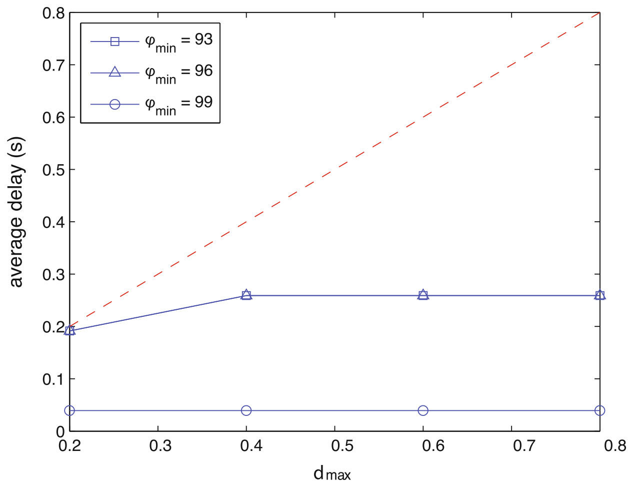

Fig. 11 Optimization of the listen and sleep times subject to delay and reliability constraints as obtained by problem (1) and the method of Sect. 7. The *figure* reports the average delay as obtained by simulations when nodes implemented the optimal solution of problem (1). The delay is reported as function of delay requirements $d_{\max} = 0.2, 0.4, 0.6, 0.8$ s, $\tau_{\max} = 95$ % and reliability requirements $\phi_{\min} = 93, 96, 99$ % for a traffic rate of $1/\lambda = 30$ s and 8 nodes. The *dashed line* denotes the border of the feasible region for the delay. The fact that the continuous lines are below the dashed line means that the optimization problem (1) provides us with optimal listen and sleep times that respect the imposed delay constraint

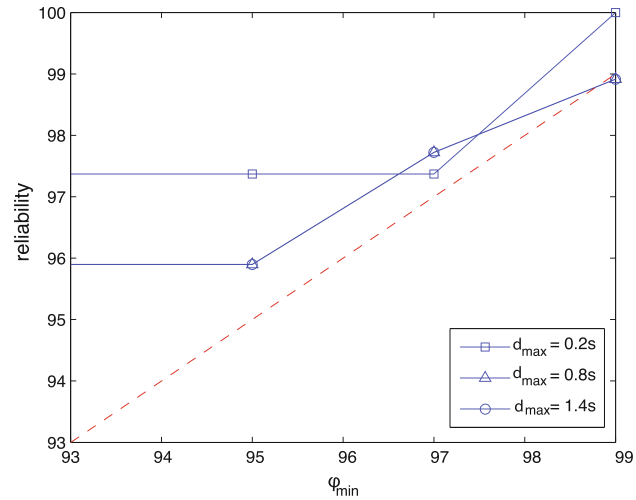

Fig. 12 Optimization of the listen and sleep times subject to delay and reliability constraints as obtained by problem (1) and the method of Sect. 7. The *figure* reports the reliability as obtained by simulations when nodes implemented the optimal solution of problem (1). The reliability is reported as function of delay requirements $d_{\max} = 0.2, 0.8, 1.4$ s, $\tau_{\max} = 95$ % and reliability requirements $\phi_{\min} = 93, 95, 97, 99$ % for a traffic rate of $1/\lambda = 30$ s and 8 nodes. The *dashed line* denotes the border of the feasible region for the reliability. The fact that the *continuous lines* are *above* the *dashed line* means that the optimization problem (1) provides us with listen and sleep times that respect the imposed reliability constraint

Figure 12 shows the reliability as a function of delay requirement $d_{\max}$ = 0.2, 0.8, 1.4 s, $\tau_{\max}$ = 95 %, and the reliability requirement $\phi_{\min}$ = 93, 95, 97, 99 %. The simulation results confirm the validity of the optimization because the requirement on the reliability are satisfied in all the cases. Note that the reliability requirement $\phi_{\min}$ = 99 % is slightly below 99 % when $d_{\max}$ = 0.8, 1.4 s due to the Euclidean distance method to approximate the estimated loss and busy channel probabilities. It is interesting to observe that the strict delay requirement $d_{\max}$ = 0.2 s determines that the dominant constraint of the optimization problem is the delay, whereas the reliability requirement $\phi_{\min}$ = 93, 95, 97 % makes the reliability constraint inactive.

We conclude that the listen and sleep time computed by our modeling and optimization allow packets to meet the delay and reliability requirements set by the application.

Finally, we remark that a comparison with constrained duty-cycling optimizations as those proposed in [19–28], is immediate, since the algorithms proposed therein assume perfect packet reception and do not consider the random MAC, which, as we have shown by analysis and simulations, cannot be neglected.

8.2 Unconstrained optimization example

We compare the minimization of (1) as obtained by the method described in Sect. 7 to the one provided by X-MAC [14]. Recall that such a protocol does not take into account random back-off, delay and reliability constraints and thus makes an unconstrained optimization of the energy consumption. Therefore, for the sake of comparison of the protocol proposed in this paper and X-MAC, we pose $d_{\max} = \infty$, $\tau_{\max} = 1$, and $\psi_{\min} = 0$, which implies neglecting the delay and reliability requirements, i.e. the energy is minimized without constraints, as done in X-MAC.

Figure 10 shows the energy consumption corresponding to the optimal protocol parameters determined by the proposed method and X-MAC. The proposed method outperforms X-MAC in all the scenarios considered. Specifically, when the packet generation period is high (300 s) the difference between X-MAC and the proposed method is small (5 % less than X-MAC), but as the packet generation period decreases the improvement is substantial, more than 50 %. The main reason for this difference is that the nodes consume much less energy in packet transmission compared to the model in [14]. X-MAC is based on the assumption that the transmitter sends preamble packets back to back until the receiver wakes up, while actually there is random back-off before packet transmissions during which the transmitter puts its radio in sleep mode. Since the transmit energy dominates the receive energy much earlier according to the model in [14], the optimal wake time becomes considerably higher compared to the actual optimal wake time that we achieve.

9 Conclusions

We propose a novel method for the optimization of the duty-cycle parameters in preamble sampling based asynchronous duty cycle protocols based on the accurate modeling of the delay, reliability, and energy consumption as a function of listen time, sleep time, traffic rate and MAC protocol parameters. The novel statistical modeling is used to tackle the challenges of the random access MAC and the sleep policy of the receivers. The results are validated through Monte Carlo simulations. The expressions of the delay, reliability and energy consumption as a function of the relevant parameters of the network are used in the unconstrained minimization of the energy consumption of the network and constrained minimization subject to delay and reliability requirements. The proposed optimization approach is demonstrated to provide significant reduction of the energy consumption compared to the previously proposed protocols in the literature.

In the future, we will focus on the extension of our theoretical analysis to the hybrid random-access/TDMA MAC, and on the investigation of the interaction of duty cycle with routing.

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